Modeling with Geometry – Examples

This essay assumes an understanding of the essay titled "Introduction to Modeling".

Examples presented here will illustrate the process of constructing a mathematical model using simple geometry facts. The resulting equations (models) will not be solved for the three reasons listed below.

1. The algebraic techniques required to solve some of the equations may be unknown to some students.
2. The algebraic process of solving the equation will distract some students from the technique of building the model.
3. The student should learn that building a model is independent of the ability to solve the model.

Example 1: If a cylinder is 15 ft. tall and its surface area, including the two ends, is 458 sq. ft., what is its height?
Analysis: The formula $S = 2\pi r^2 + 2\pi rh$ is applicable. Substitute 458 for S and 15 for h to obtain $458 = 2\pi r^2 + 30\pi r$. This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 2: What is the height of a cone shaped can with radius 2.5 inches and volume 40 cu. in.?

Analysis: The formula for a cone $V = \frac{1}{3}\pi r^2 h$ is appropriate.

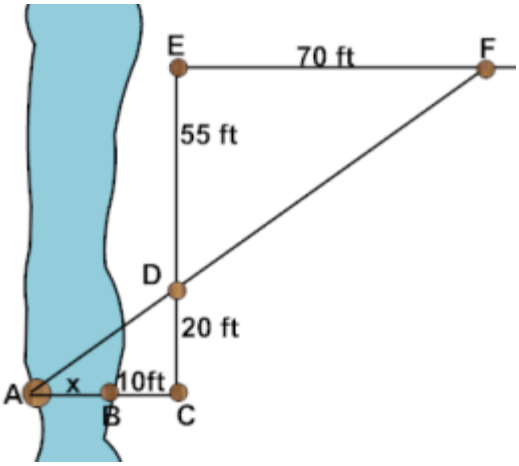
Substitute 2.5 and 40 into the formula to obtain $40 = \frac{1}{3}\pi(2.5)^2 h$

This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

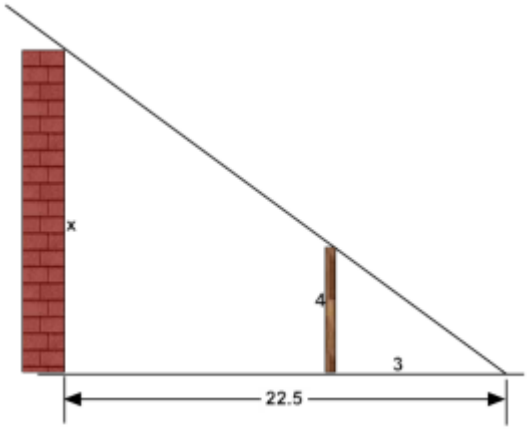
Example 3: Consider the drawing at the right. Calculate the width x of the river.
Analysis: Triangles ACD and DEF are similar triangles.

Therefore $\frac{x+10}{20} = \frac{70}{55}$.

This equation is the mathematical model. The solution set for this equation will be the answer to the original question.



Example 4: A post 4 feet tall makes a 3-foot shadow. If the shadow of a brick wall is 22.5 feet long, how tall is the wall?



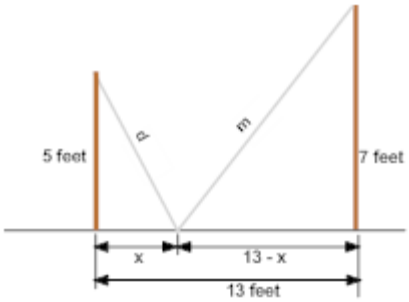
Analysis: Refer to the drawing at the right. Note the similar triangles. Use them to set up the equation.

$$\frac{x}{22.5} = \frac{4}{3}$$

This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 5: Two vertical poles are 13 feet apart. One pole is 5 ft tall the other is 7 feet tall. A 20 ft. cable is to be attached to the top of each pole and anchored to the ground between the posts so that it acts as a support for both poles. What will be the distance from the 5 ft. pole to the anchor.

Analysis: Because this problem is geometric in nature, a sketch is appropriate. Notice that in the sketch we have identified and assigned a variable to the length we are asked to find. We have incorporated into the sketch as much detail as we are able to glean from the statement of the problem.



The statement of the problem tells us that **d + m** = 20. The obvious right triangles cause us to recall the **Pythagorean Theorem:**

> If a and b are the legs of a right triangle with hypotenuse c, then $a^2 + b^2 = c^2$

A little deductive reasoning, gives us the two specific Pythagorean statements:

$d = \sqrt{x^2 + 5^2}$ and $m = \sqrt{(13 - x)^2 + 7^2}$

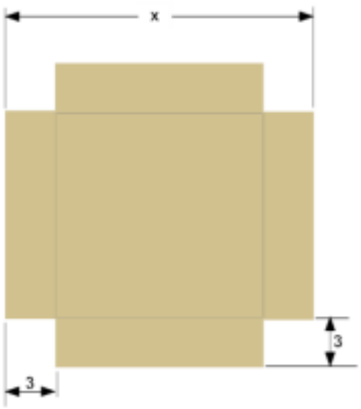
From which we get $\mathbf{d + m = \sqrt{x^2 + 5^2} + \sqrt{(13 - x)^2 + 7^2}}$

The blue highlighting shows that we now have two expressions for the quantity d + m. Therefore those two expressions must be equal (Transitive Property). This observation yields $\sqrt{x^2 + 5^2} + \sqrt{(13 - x)^2 + 7^2} = 20$

This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

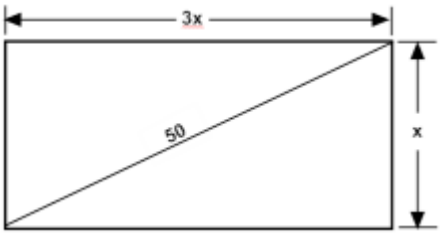
Example 6: An open box is to be made from a square sheet of cardboard by cutting out squares from each corner and then folding up the sides. The box must have a volume of 300 cubic inches. What must be the dimensions of the original sheet of cardboard?

Analysis: The diagram at the right is helpful.



Let x be the length of a side of the original square of cardboard.
Then the box will be 3 inches deep with both length and width x – 6 inches.
The volume of the box will be (x - 6)(x – 6)(3)
The volume of the box will be 300.
We now have two expressions for the same quantity
Therefore (by the Transitive Property) the two expressions must be equal.
This observation yields (x - 6)(x – 6)(3) = 300.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 7: A rectangle is three times longer than it is wide. Its diagonal is 50 inches long. What are the dimensions of the rectangle? What is its perimeter? What is its area?



Analysis: Consider the sketch at the right.
The Pythagorean Theorem comes to mind as a perfect model. When the Pythagorean Theorem is applied to this drawing we obtain $50^2 = x^2 + (3x)^2$
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.