Modeling Work Problems – Examples

This essay assumes an understanding of the essay titled "Introduction to Modeling".

Examples presented here will illustrate the process of constructing a mathematical model for so called "rate problems". The resulting equations (models) will not be solved for the three reasons listed below.

1. The algebraic techniques required to solve some of the equations may be unknown to some students.
2. The algebraic process of solving the equation will distract some students from the technique of building the model.
3. The student should learn that building a model is independent of the ability to solve the model.

Percent problems are rate problems but are given their own category simply because they cause so much difficulty for students and society in general. There are only three possibilities for percent problems:

i. Given percent and base, find the percentage.
ii. Given percent and percentage, find the base.
iii. Given percentage and base, find percent.

In all three cases the familiar formula

Percentage = (Percent)(Base)

is applicable. For the sake of brevity, we will shorten that formula by thinking of percentage as the Amount and will use single letters to quote the formula. So the formula takes on the appearance

A=PB

Regardless of the percent problem, substitute the two know quantities into this formula to obtain the mathematical model.

Example 1: What is 47% of 238?
Analysis: Change the percent 47% to 0.47. Identify 238 as the base. Substitute into A = PB to obtain A = (0.47)(238).
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 2: 238 is 47% of what number.
Analysis: Change the percent 47% to 0.47. Identify 238 as the percentage.

Substitute into A = PB to obtain 238 = (0.47)B.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 3: 238 is what percent of 47?
Analysis: Identify 47 as the base. Identify 238 as the percentage.
Substitute into A = PB to obtain 238 = 47P.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 4: One hundred ounces of juice that is 50% tomato juice is added to 200 oz of a vegetable juice that is 25% tomato juice. What is the percent concentration of tomato juice in the resulting mixture?
Analysis: The total amount of the final mixture is 300 oz.
The amount of tomato juice contributed by the 50% mixture is 50 oz.
The amount of tomato juice contributed by the 25% mixture is 50 oz.
The total amount of tomato juice in the final 300 oz mixture is 100 oz.
The question now is a standard percent problem.
100 is what percent of 300?
Identify 300 as the base. Identify 100 as the percentage.
Substitute into A = PB to obtain 100 = 300P.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 5: A man had $12,000 to invest. He invested $8,500 of it at 4% and the rest at such a rate that his total income from the two investments was $575 annually. At what rate was the second sum invested.
Analysis: Let x be the rate of the second investment.
The amount invested at this rate was $3,500.
The total interest earned was $575.
The total interest earned was (8500)(0.04) +(3500)x
We now have two expressions for the same quantity.
Therefore (by the Transitive Property) these two expressions must be equal.
This observation yields (8500)(0.04) +(3500)x = 575.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 6: Jane invested one part of her $25,000 at 5.5% per year and the other part at 4.5% per year. Her income from the two investments was $760. How much did she invest at each rate?
Analysis: Let x be the amount invested at 5.5%.

Then 25000 - x is the amount invested at 4.5%.
The total interest earned was 760.
The total interest earned was (25000 – x)(0.045) + (0.055)x
We now have two expressions for the same quantity.
Therefore (by the Transitive Property) these two expressions must be equal.
This observation yields (25000 – x)(0.045) + 0.055x = 760.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.

Example 7: In an election contested by two parties, candidate Jones secured 12% of the total votes more than candidate Smith. If Smith got 132,000 votes, by how many votes did it lose the election?
Analysis: Let x be total number of votes cast.
Smith got 132,000 votes
Then Jones got 132,000 + .12x votes.
The total number of votes cast was 132,000 + (132,000 + 0.12x)
We now have two expressions for the same quantity.
Therefore (by the Transitive Property) these two expressions must be equal.
This observation yields 132,000 + (132,000 + 0.12x) = x.
This equation is the mathematical model. The solution set for this equation will be the answer to the original question.